Learning path for patent examiners

Mathematics and its applications: Intermediate level

Version: May 2024

Introduction

This publication, "Mathematics and its applications, Intermediate level", is part of the "Learning path for patent examiners" series edited and published by the European Patent Academy. The series is intended for patent examiners at national patent offices who are taking part in training organised by the European Patent Office (EPO). It is also freely available to the public for independent learning.

Topics covered include novelty, inventive step, clarity, unity of invention, sufficiency of disclosure, amendments and search. Also addressed are patenting issues specific to certain technical fields:

- patentability exceptions and exclusions in biotechnology
- assessment of novelty, inventive step, clarity, sufficiency of disclosure and unity of invention for chemical inventions
- the patentability of computer-implemented inventions, business methods, game rules, mathematics and its applications, presentations of information, graphical user interfaces and programs for computers
- claim formulation for computer-implemented inventions

Each publication focuses on one topic at entry, intermediate or advanced level. The explanations and examples are based on the European Patent Convention, the Guidelines for Examination in the EPO and selected decisions of the EPO's boards of appeal. References are made to the Patent Cooperation Treaty and its Regulations whenever appropriate.

The series will be revised annually to ensure it remains up to date.

Disclaimer

This publication is for training and information purposes only. Although it has been prepared with great care, it cannot be guaranteed that the information it contains is accurate and up to date; nor is it meant to be a comprehensive study or a source of legal advice. The EPO is not liable for any losses, damages, costs, third-party liabilities or expenses arising from any error in data or other information provided in this publication.

The opinions expressed in this publication are not necessarily those of the EPO.

All references to natural persons are to be understood as applying to all genders.

Contents

Legal references

1. Learning objectives

Participants to this course will learn:

- The definition of Mathematical Method and Artificial Intelligence
- To apply the two-hurdle approach for a MM and AI application
- The two dimensions in which features of a MM can contribute to the technical character of an invention
- The requirement of a specific technical purpose
- The requirement of a specific technical implementation and technical considerations regarding the internal functioning of the computer system

2. What is a "mathematical method"?

A mathematical method is a method that has steps that are mathematical computations or of an algorithmic nature, sometimes using formulae to describe what is computed.

Given the key role of mathematics in technology in general, and that of AI/ML in particular, there are specific criteria for determining whether mathematical features in a claim contribute to technical character. A distinction has to be drawn between claims in which mathematics is claimed "as such" versus claims in which a mix of technical and non-technical features is present: "mixed-type" inventions. While mathematical methods are non-inventions when claimed "as such" (Article 52(a) and (3) EPC), they are at the core of technology and may well play a decisive part in achieving technical effects (G-II, 3.3).

The Fourier transform is an example of a mathematical method without which much technological progress would never have taken place. It is used in fields such as signal processing, optics and the analysis of protein structure.

Examples

Another example of a mathematical transform at the core of technological progress is the Radon transform, which enabled computer tomography once imaging devices became available.

Legal references:
Art. 52(2) EPC, Art. 52 (3) EPC

3. First hurdle for mathematics

The two-hurdle approach harmonises the interpretation of the exclusions by setting up two hurdles to be passed – the first, avoiding the exclusion from patentability, that is the claimed invention being eligible for patenting (Article 52(2) and (3) EPC), and the second, the claimed invention not being obvious when solving a technical problem (Article 56 EPC).

The first hurdle, also referred to as the patent eligibility hurdle, requires that the claimed subject-matter as a whole must not fall under the "non-inventions" defined in Art. 52(2) and (3). The exclusion from patentability of the subject-matters and activities referred to in Art. 52(2) is limited by Art. 52(3) to such subject-matters or activities that are claimed "as such". This limitation is a bar to a broad interpretation of the non-inventions. It implies that one technical feature is sufficient for eligibility: if the claim comprises at least one feature which does not fall under the "non-inventions" defined in Art. 52(2), it is not excluded "as such". This assessment is made without reference to the prior art.

For example, if the claimed subject-matter is directed to or uses **technical means**, it is an invention within the meaning of Art. 52(1).

A claim that is directed to a mathematical method can require technical means explicitly (typically "computer-implemented") or implicitly. Technical means may be implicit when steps of a method necessarily require the use of technical means. For instance, a step of "collecting/measuring an electric signal" requires technical means as it cannot be performed mentally.

A claim that is "pure maths" with no technical means involved will be objected to as being excluded from patentability under Article 52(2) and (3) EPC. However, Article 52(2) EPC is a non-exhaustive list of non-inventions, and an abstract mathematical object (a polyhedron, for example, or a non-computer-implemented data structure or a trained neural network) can also be considered a non-invention under Article 52(1) EPC.

If some sort of mathematics is performed on technical or physical parameters but again with no technical means involved, the claim will be objected to under Article 52(2)(c) and (3) EPC because it represents a mental act "as such" – the claim can be taken to be mere calculation instructions without using technical means.

Importantly, the fact that the parameters of a mathematical method have technical meaning does not imply the use of technical means. Neither does the complexity of the method imply technical means (G-II, 3.5.1), nor that the skilled person would find it obvious to use technical means.

Legal references:
GL,G-II, 3.3, GL,G-II, 3.5.1

4. Mathematics: the two dimensions of the second hurdle

Once the first hurdle is passed, the second hurdle is examined. The second hurdle is where inventive step is assessed.

When assessing inventive step (Article 56 EPC), i.e. the requirements of the second hurdle, it must be ascertained which features of the claimed subject-matter contribute to technical character as only these may support the presence of an inventive step.

Section G-VII, 5.4 of the Guidelines sets out the problem-solution approach for claims comprising technical and non-technical features. All features contributing to the technical character are taken into account for assessing inventive step.

If the invention comprises a mathematical method, the question arises as to whether any features of the claimed method defining this mathematical method contribute to the technical character of the invention, and if so, which ones do so.

Section G-II, 3.3 specifies when a mathematical method (or mathematical method steps) can contribute to the technical character of the invention and thus needs to be taken into account when assessing inventive step in combination with the technical features.

There are essentially two orthogonal "dimensions" along which a mathematical method may make this kind of technical contribution (G-II, 3.3).

A mathematical method may contribute to the technical character of an invention, i.e. contribute to producing a technical effect that serves a technical purpose by its application to a field of technology and/or by being adapted to a specific technical implementation.

For technical applications, when assessing the contribution made by a mathematical method to the technical character of an invention, it must be taken into account whether the method, in the context of the invention, contributes to the technical solution of a technical problem by providing a technical effect, at least implicitly, across substantially the whole scope of the claim, i.e. in all relevant embodiments.

For technical implementations, a computer-implemented mathematical method may also contribute to the technical character of the invention independently of any technical application when the claim is directed to a specific technical implementation of the mathematical method and the mathematical method is particularly adapted for that implementation in that its design is motivated by technical considerations regarding the internal functioning of the computer.

When a claim is directed to a computer program, a further technical effect is required (G-II, 3.6) because running a program on a computer always has the – trivial – effect of moving electrons in the computer's circuits or accessing the computer's memory, for example. Normally, this further technical effect results from the corresponding computer-implemented method itself producing the further technical effect. If it is established that the claim as a whole produces a technical effect due to having been applied to a field of technology and/or adapted to a specific technical implementation, and that the mathematical method contributes to this effect, technical effects of distinguishing features, such as improved processing speed, can indicate the presence of an inventive step.

Note that contributing to technical character by generating a technical effect in the sense of the EPC (for the purposes of Article 56 EPC) is to be distinguished conceptually from being "technical" in the more common sense of belonging to a field of technology (in the sense of Article 52(1) EPC). An invention can pertain to a field of technology without necessarily achieving a technical effect. For example, if a claim encompasses relevant non-technical uses of generated data resulting from a simulation (such as gaining scientific knowledge about a technical system), a technical effect is not achieved even if the simulated system belongs to a field of technology (G-II, 3.3.2).

Legal references:
GL,G-II, 3.3, GL,G-II, 3.6, G 1/19, GL,G-VII, 5.4

Mathematics: the first dimension of the second hurdle

For the dimension of technical application, as mentioned above the relevant question is whether the mathematical method provides a technical effect serving a technical purpose.

The most common situation is when the claim explicitly or implicitly specifies how the output is used. In that case, usually all that needs to be determined is whether this use is technical. However, there are other cases where the technical effect does not rely on the use of the output of the method.

A claim should be functionally limited to its purpose, whether explicitly or implicitly. Additional specifications as to how the input and output relate to the purpose are normally necessary to establish the contribution of the mathematical steps to technical character, so that the mathematical method is causally linked to a technical effect.

For example, a "computer-implemented method for classifying records comprising mathematical steps, the classified records being used in a billing procedure" serves a business purpose, not a technical purpose. Automatically classifying data records serves merely to classify the data records without implying any technical use of the classification.

The purpose should be specific, i.e. not generic and *pro forma*, e.g. "controlling a technical system".

Specifying that the input to the mathematical method is measured data/physical data it does not necessarily imply that the mathematical method contributes to the technical character of the invention. On the other hand, if the mathematical method has technical input (for example from a physical measurement using technical means) and contributes to an indirect measurement of the physical state of an existing physical entity, this may provide a technical effect.

Legal references:
G-II, 3.3

Mathematics: the second dimension of the second hurdle

A mathematical method may contribute to the technical character of an invention if its design is motivated by technical considerations relating to the internal functioning of the computer. This may happen if the mathematical method is designed to exploit particular technical properties of the technical system on which it is implemented to bring about a technical effect such as efficient use of computer storage capacity or network bandwidth. However, any such implementation details concerning hardware would have to be disclosed in the patent application and should appear as limiting features in the claims.

An example might be assigning the execution of data-intensive training steps of a machine-learning algorithm to a graphical processing unit (GPU) and preparatory steps to a standard central processing unit (CPU) to take advantage of the parallel architecture of the computing platform.

In another example, "a mathematical method for distributing processing load in a computer network" may provide a technical effect serving a technical purpose. If the actual processing load in the computer network is dependent on the mathematical method, the mathematical method contributes to a technical effect solving a specific technical problem relating to the method's implementation.

In yet another example, countermeasures against power analysis attacks, such as including masking operations, may have a technical character since they change the computer's operation based on a technical understanding of its internal functioning. Power analysis attacks involve measuring the power consumption during cryptographic operations. The encryption key may be found by performing a statistical analysis on the measurements. If the claim defines masking operations that do not change the result of the cryptographic computation, i.e. the output of the cryptographic method stays the same, the result of the masking operations does not affect the output of the mathematical method, but is meant to protect the cryptographic operation against an attack. The operation of the computer is modified so that its power consumption differs from what would be observed with only the computation to be protected. The mathematical step of adding masking operations exploits the technical property of power consumption of the computer to prevent deciphering the encryption key.

Normally, a computer-implemented method which is hardware-independent is unlikely to provide a technical effect in terms of the second dimension of the second hurdle (technical implementation). If an effect occurs independently of the hardware used (or even when no hardware is used, but the method is executed as a mental act), it is unlikely that the design of the mathematical method is motivated by technical considerations of the internal functioning of the computer system or network. Thus, an adaptation to particular hardware or the adaptation of the algorithm to specifically suit the properties of the hardware is needed.

Improved computational efficiency of a mathematical method is not in itself a technical effect, unless it is linked to the particular hardware executing the mathematical method, or unless another further technical effect is achieved (G-II, 3.6), for example by the method's output. In any case, a claimed computational speed-up must be credibly achieved in substantially all embodiments (e.g. using any technical means allowed by the claim, and using any allowed parameter set). This may be difficult to prove for a hardware-independent method.

Legal references:
GL,G-II, 3.3

5. What is artificial intelligence (AI)?

AI is a broad sub-field of computer science that covers many different computational models for solving data analysis problems. The focus of this section is on the machine learning branch of AI, but AI also includes other branches such as symbolic AI which are present in what are known as "expert systems".

AI/ML algorithms/models are supported by advanced mathematics and are themselves of an abstract mathematical nature. This is why the topic of AI and machine learning is subsumed under mathematical methods in the Guidelines.

There are four main data analysis problems tackled by non-symbolic AI: classification, clustering, regression and dimensionality reduction.

- Classification is about identifying the category of a new observation on the basis of a labelled training set with categorised observations by which a classifier (a classification algorithm/model) has been trained. Examples of classification are identifying a file as being infected with a virus (according to certain characteristics) or determining whether to buy or sell stock in real-time depending on the current stock chart and a training set from top-trader past activities.
- Clustering aims at grouping sets of (data) objects such that objects in one cluster are more similar to each other than those in another cluster. An example of clustering is identifying gene families in a DNA sample.
- Regression is estimating a relationship from among variables of a dataset, typically by fitting a curve to the dataset.
- Dimensionality reduction attempts to reduce the number of variables characterising a dataset while retaining (at least some of) its information content.

The basic purpose of models tackling classification, clustering, regression, dimensionality reduction, etc. is abstract. The fact they are "trained" by a training algorithm does not change this (G-II, 3.3.1).

Caution is required in the field of AI/ML due to the use of field-specific terminology that may sound like it implies technical means ("machine" in support vector machine or "network" in neural network) but in fact generally refers to abstract models devoid of technical character (G-II, 3.3.1).

The take-home message here is that the principles of examination regarding mathematical methods also apply to AI and ML.

Legal references:
GL,G-II, 3.3.1

6. AI: the two dimensions of the second hurdle

Like with mathematical methods in general, the issue for methods involving AI is also whether the AI and ML method (or method steps) contributes to the technical character of the invention.

First dimension – technical applications

The AI and ML method (or method steps) contributes to the technical character of the invention to the extent that, in the context of the invention, a technical purpose is served by the technical application of the (overall) method, i.e. to solve a technical problem in a technical field by providing a technical effect, at least implicitly, in substantially all embodiments.

Prominent examples of technical applications of AI/ML are found in the fields of speech/image processing, fault detection and engine control.

Classification for determining the price of a service is a typical business application, not a technical application.

The requirements of G-II, 3.3 apply.

Example

An enhanced classifier for classifying digital images on the basis of an expanded training set

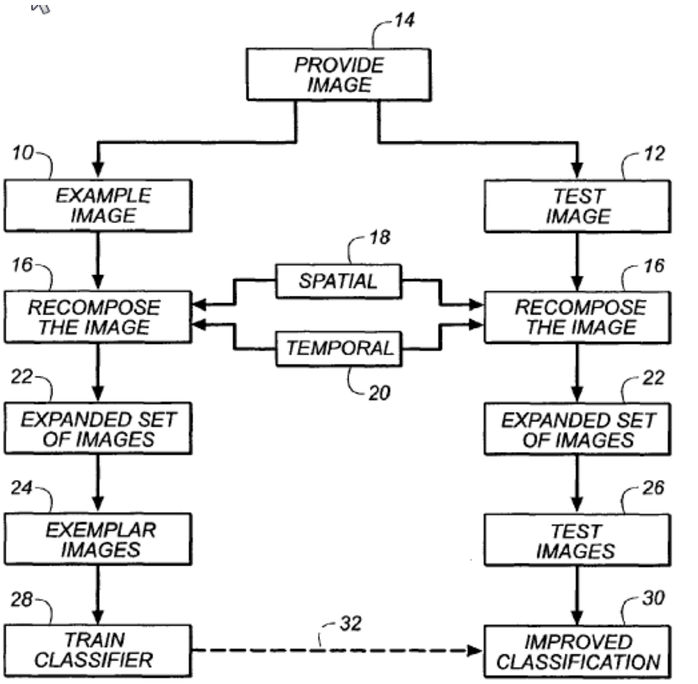


FIG. 9

This example is based on case T 1286/09 in the field of image classification.

The purpose in this case was to classify pictures as "sunset", "beach", "forest", "picnic" or the like. In the prior art, doing so was found to be difficult because there were a huge variety of "sunset" images.

In this invention, an improved classifier was implemented by adapting the training data. Given a set of images for training, the set was first expanded by varying the colour codes slightly and gradually, to produce more training data.

In the end, the training dataset was expanded, and the resulting better trained classifier was found to perform much better and to be much more reliable.

Legal references:
GL G-II, 3.3

7. Simulation, modelling and design

A "simulation" is an approximative imitation of the operation of a system or process on the basis of a model of that system or process. This may happen on the basis of equations describing, for example, the temporal evolution (the operation in a narrow sense), but also by predicting merely the result of the operation (in a more general sense). Likewise, the model may be a physics-based model or a generalised model (e.g. a machine-learning model).

A "model" or "design" is a description of a system or process. This may take the form of equations or of geometric relationships, or of other generalised relations, usually of mathematical nature. The model or design may be three-dimensional (e.g. a rigid body model, a surface or volume mesh or a solid body model), or it may be a different representation, such as a blueprint or another form of description (e.g. an RTL model of a circuit).

Computer-implemented methods of simulating, designing or modelling are examined according to the same criteria as any other computer-implemented inventions (G-VII, 5.4, G 1/19).

In particular, the mathematical steps relating to simulation, modelling or design must interact with the claim's technical features in **contributing** to the technical solution of a technical problem by providing a technical effect, at least implicitly, across substantially the whole scope of the claim, i.e. in all relevant embodiments.

Such a technical effect may be achieved, for example, in one of the following manners (G-II, 3.3.2, G 1/19):

- by providing technical output having a specific technical application or being limited to an intended technical use (first dimension)
- by interacting with the executing computer system via a technical implementation (second dimension)
- by interacting with physical reality at the onset or throughout its execution via technical input

This list is not exhaustive, and other technical effects may be acknowledged as technology progresses. The different options are discussed in more detail in the advanced course.

For the purposes of establishing a model and formalising it through equations, it is likewise irrelevant whether a specific physically-existing technical entity (e.g. a particular building) is modelled or simulated or whether the represented object is itself abstract (G 1/19, Section 109). In any case, the model or design is not to be equated with the real entity, but merely constitutes an abstract or virtual representation of it. Establishing the model and the equations is in general a purely mental act, even though these activities might be supported by computers (G 1/19, Section 112).

For establishing the presence of a technical effect, it is not decisive whether the modelled, designed or simulated system or process belongs to a field of technology (G 1/19, Section 106, Section 121), whether the simulation or model reflects technical principles underlying the simulated system nor how accurately it does so. Rather, it is relevant whether the simulation of the system or process contributes to the solution of a technical problem by providing a technical effect in substantially all embodiments, at least implicitly.

Any technical considerations must pertain to the invention, i.e. to the further use of the model/simulation or to its technical implementation. The technical considerations which may be required in order to understand the simulated/modelled system or process are not necessarily relevant to whether the invention solves a technical problem by producing a technical effect (G 1/19, Section 125).

Legal references:
GL,G-II, 3.3.2, G 1/19

8. Mathematics – examples

Example 1 — first hurdle

An example with the Fast Fourier Transform and typical concrete claim wording comprising mathematical features is provided below. There is no need to go into the mathematical detail.

> A method for calculating a Fast Fourier Transform y of vector x, wherein y = F_n * x,
>
> where $F_n = \begin{pmatrix} & \cdots & \\ \vdots & \ddots & \vdots \\ 1 & \cdots & \omega^1 \end{pmatrix}$ and y is calculated by calculating u = $F_{n/2} * \begin{bmatrix} x_0 \\ x_2 \\ . \\ x_{n-2} \end{bmatrix}$ and v =
>
> $F_{n/2} * \begin{bmatrix} x_1 \\ x_3 \\ . \\ x_{n-1} \end{bmatrix}$ and calculating each y_i using $y_i = \begin{cases} u_i + \omega_n^i v_i \; for \; 0 \leq i < n/2 \\ u_{i-n/2} + \omega_n^i v_{i-n/2} \; for \; n/2 \leq i < n \end{cases}$

It should first be noted that no computer-implementation is claimed, so no technical means are involved.

The method manipulates purely abstract input parameters, i.e. the vector x, the matrices F_n, $F_{n/2}$ and the resulting vector y.

The method concerns an abstract mathematical method **as such**, executed without technical means. An objection should thus be raised under Article 52(2)(a) and (3) EPC.

Example 2 — first hurdle

In the slightly modified version of the previous claim below, it is added that x represents a series of temperature measurements. Therefore, x is no longer an abstract parameter. In other words, the method is already mathematics "applied" to concrete physical parameters.

> A method for calculating a Fast Fourier Transform y of vector x, comprising
>
> … *[formulae]* …
>
> wherein x represents a series of temperature measurements.

Since no technical means are involved and it is thus still possible to construe the claim as an instruction to the human mind on how to calculate a transform, an objection under Article 52(2)(c) and (3) EPC is raised (the claimed method encompasses a mental act as such). This illustrates that specifying only the input parameters of a mathematical method does not necessarily overcome the first hurdle.

Example 3 — first hurdle

With additional restrictions compared with the previous claim (see below), with the measurements now being obtained using a sensor, the claim requires technical means and is no longer excluded from patentability.

> A method for calculating a Fast Fourier Transform y of vector x, comprising
>
> - obtaining a series of measurements **using a temperature sensor**
> - assigning the obtained series of measurements to x
> - calculating y using
>
> ... *[formulae]* ....

Note that passing the first hurdle does not imply anything about the second hurdle. This is determined by whether the mathematical method **interacts** with the technical input and thus contributes to the technical effect of performing a temperature measurement using sensors.

Example 4 — the first dimension of the second hurdle — simulation method

The invention concerns a computer-implemented method for the virtual testing of a virtual weldment (T 2594/17).

Claim 1 comprises the step of "an analysis engine configured to perform **simulated testing of a 3D virtual weldment** (2200), and further configured to perform inspection of at least one of a 3D virtual weldment (2200) before simulated testing, a 3D animation of a virtual weldment (2200) under simulated testing, and a **3D virtual weldment** (2200) after simulated testing for at least one of pass/fail conditions and defect/discontinuity characteristics; … wherein said **simulated testing** includes at least one of simulated destructive testing and simulated nondestructive testing of the **virtual weldment**."

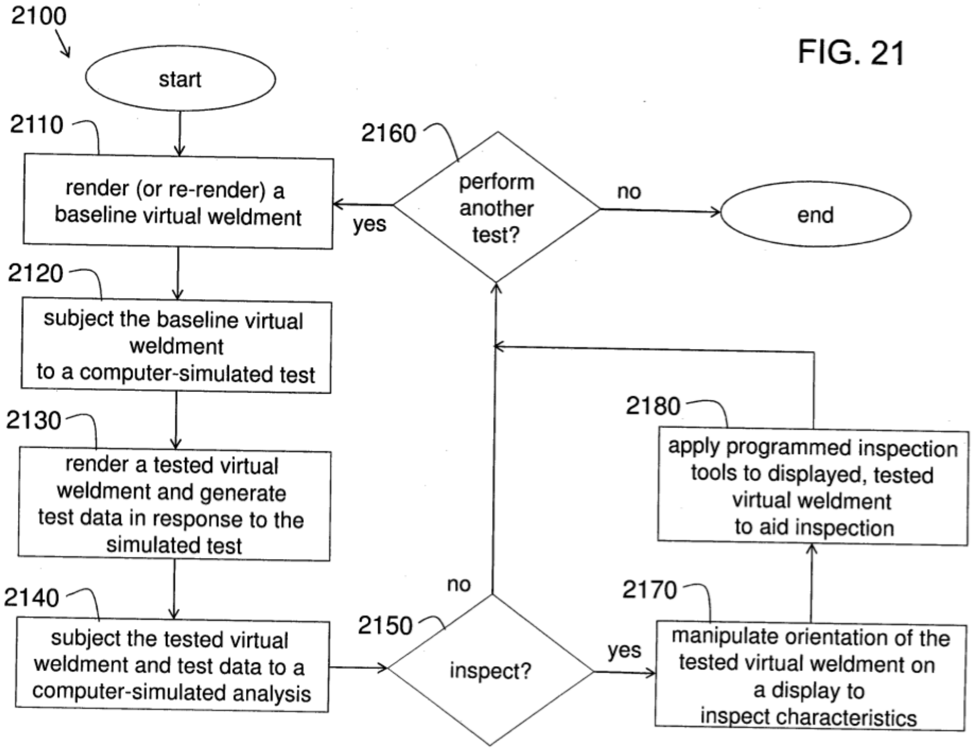


FIG. 21

The mathematical method operates merely on a "virtual weldment" which need not exist in physical reality. The claim merely generates data which is not functionally limited to an inherently technical use but may be used for a variety of non-technical purposes, for example for training purposes. The board identified the computer implementation as the only technical feature and took the closest prior art to be a known general-purpose computer (R.3.1.3). The board held that the mathematical steps do not interact with the technical features of the claim (R.3.2.9) and thus do not contribute to the technical character of the invention; they merely perform image processing in order to generate cognitive data (R.3.2.5 and R.3.2.11). The mathematical method is hence given to the skilled person as part of the problem to be solved (automating the mathematical method). The solution to this problem is straightforward, and merely requires routine programming skills (G-VII, 5.4).

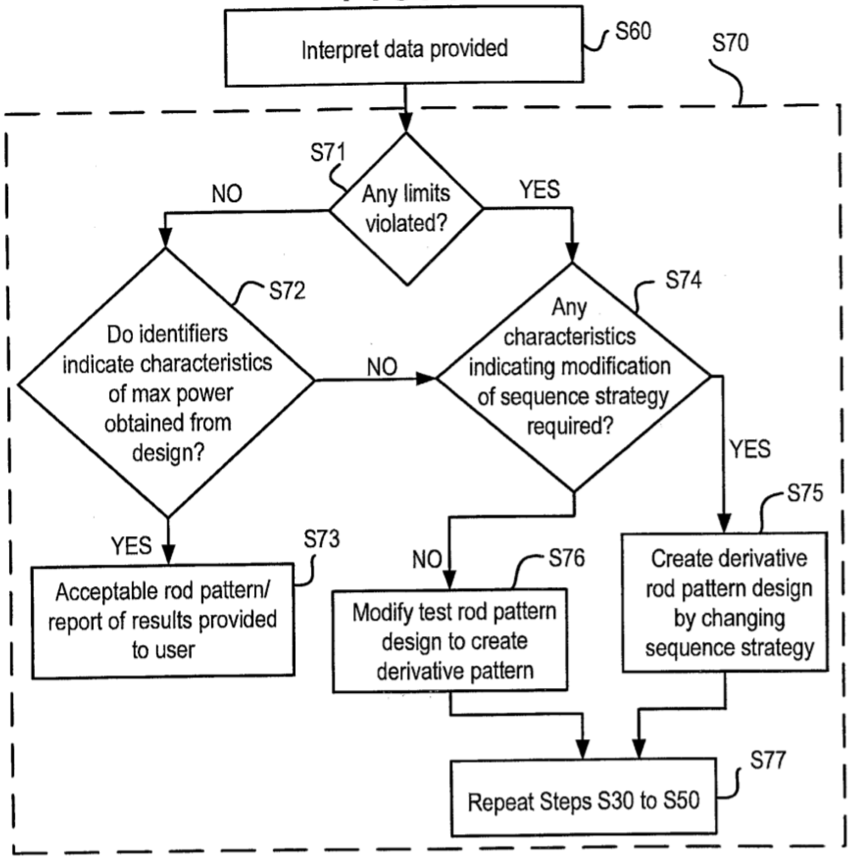
This example shows that a known general-purpose computer can be used as closest prior art, as foreseen in G 1/19, Section 79.

Example 5 — the first dimension of the second hurdle

The invention concerns a power supply method (T 1766/16). Compared to the methods of A3 and A7, the claimed power supply method is distinguished by the addition of a mathematical step for determining the program period from the variation of the instantaneous power.

6/16

FIG. 7A



The board held that in the absence of any details as to the function linking the program period and the power variation, this may be arbitrary, for example monotonically increasing, as in the embodiment disclosed in the application, but also monotonic decreasing. It is therefore impossible to determine any technical effect of the mathematical method (R.18), which effect would solve any particular technical problem. The effects which are mentioned in the application are not credibly achieved in substantially all embodiments.

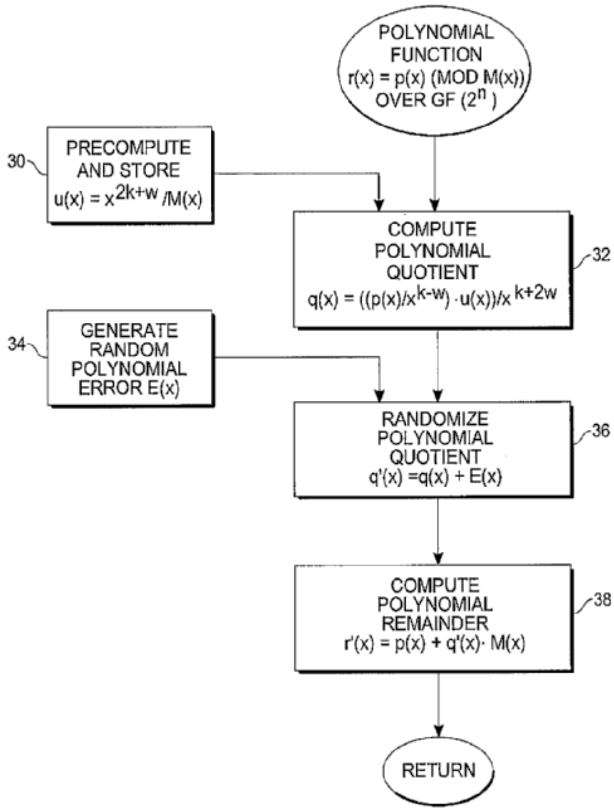
The broad wording of claim 1 covers a wide range of processes whose objective would be to smooth the supplied power or, on the contrary, to limit the number of instantaneous power variations, or even to achieve other objectives which are not even mentioned in the specification (R.23).

No technical problem is solved compared to the available prior-art documents. Therefore, the subject-matter does not involve an inventive step.

In contrast to the previous example, in this example only the distinguishing feature with respect to the closest prior art is examined for a technical effect solving a technical problem. As it does not contribute to a technical effect, it does not support an inventive step.

Example 6 — the second dimension of the second hurdle

The invention concerns a "polynomial reduction operation" (T 1925/11). For a modulus of high degree (multi-word), the operation can be performed with word shifts rather than bit shifts. To this end, the formulae used are reformulated in terms of the "word size w", more precisely in terms of divisions by x**((2k+w)) and x**((k-w)).



Without going into the complex mathematical details of this example, the key message here is that the mathematical operations performed are specifically adapted to the underlying architecture that offers word shift operations.

From point 8 of the Reasons in the aforementioned case, it is clear that the board considers that the implementation of the algorithm in terms of word shifts (of the underlying hardware) contributes to inventive step, and thus implicitly to technical character. It is important to note that the claim is **implicitly limited** to using specific hardware capable of word shifts.

This is said to "simplify handling of the polynomial quantities on computational hardware".

European Patent Academy
European Patent Office
Munich
Germany

Responsible for the content
European Patent Academy
academy@epo.org