Learning path for patent examiners

Mathematics and its applications:
Advanced level

Version: October 2025

Introduction

This publication, <mark>"Mathematics and its applications, Advanced level"</mark>, is part of the "Learning path for patent examiners" series edited and published by the European Patent Academy. The series is intended for patent examiners at national patent offices who are taking part in training organised by the European Patent Office (EPO). It is also freely available to the public for independent learning.

Topics covered include novelty, inventive step, clarity, unity of invention, sufficiency of disclosure, amendments and search. Also addressed are patenting issues specific to certain technical fields:

- patentability exceptions and exclusions in biotechnology
- assessment of novelty, inventive step, clarity, sufficiency of disclosure and unity of invention for chemical inventions
- the patentability of computer-implemented inventions, business methods, game rules, mathematics and its applications, presentations of information, graphical user interfaces and programs for computers
- claim formulation for computer-implemented inventions

Each publication focuses on one topic at entry, intermediate or advanced level. The explanations and examples are based on the European Patent Convention, the Guidelines for Examination in the EPO and selected decisions of the EPO's boards of appeal. References are made to the Patent Cooperation Treaty and its Regulations whenever appropriate.

The series will be revised annually to ensure it remains up to date.

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Contents

Legal references

1. Learning objectives

Participants to this course will:
- Be given examples of technical effects and technical purposes
- Learn when to consider the argument of improved computational efficiency and/or accuracy
- Learn how to deal with inventions involving mathematical steps relating to simulation, modeling, and design
- Learn when to consider the argument of an indirect measurement

2. Mathematics: the first dimension of the second hurdle

For the dimension of technical application, as mentioned previously, the relevant question is whether the mathematical method produces a technical effect serving a technical purpose.

The most common situation is when the claim explicitly or implicitly specifies how the output is used. In that case, what needs to be determined is whether this use is technical and, if so, which of the mathematical features, if any, contribute to the technical effect in substantially all embodiments, that is, in all relevant embodiment falling under the scope of protection. However, there are other cases where the technical effect does not rely on the use of the output of the method.

A claim should be functionally limited to its purpose, whether explicitly or implicitly. Additional specifications as to how the input and output relate to the purpose are normally necessary to establish how the mathematical steps contribute to technical character.

The purpose should be specific, i.e. not generic and *pro forma*, e.g. "controlling a technical system" or "manufacturing an object".

Specifying that the input to the mathematical method is measured data/physical data does not necessarily imply that the mathematical method contributes to the technical character of the invention.

A list of example technical contributions of a mathematical method:
- controlling a specific technical system or process, e.g. an X-ray apparatus or a steel cooling process
- determining from measurements a required number of passes of a compaction machine to achieve a desired material density
- digital audio, image or video enhancement or analysis, e.g. de-noising, estimating the quality of a transmitted digital audio signal
- separating sources in speech signals; speech recognition, e.g. mapping a speech input to a text output
- encoding data for reliable and/or efficient transmission or storage (and corresponding decoding), e.g. error-correction coding of data for transmission over a noisy channel, compression of audio, image, video or sensor data
- optimising load distribution in a computer network
- encrypting/decrypting or signing electronic communications; generating keys in an RSA cryptographic system
- providing a genotype estimate on the basis of an analysis of DNA samples, and providing a confidence range for this estimate to quantify its reliability

- providing a medical diagnosis by an automated system processing physiological measurements
- determining a subject's energy expenditure by processing data obtained from physiological sensors; deriving a subject's body temperature from data obtained from an ear temperature detector.

Legal references:
G-II, 3.3

3. Mathematics: the second dimension of the second hurdle

Mathematical features can contribute to technical character through a "specific" technical implementation.

This dimension is independent, which means that when the claim is directed to a "specific" implementation of a mathematical method, no limitation to a further technical use is necessary.

For the dimension of technical implementation, there are two typical situations.

First situation – adaptation of the computer to the (known) method

This is when specific hardware is used to execute a mathematical method.

For example, a (known) method may be executed on particular hardware (e.g., a quantum-computing processor, or an analogue computer, e.g., using hydrodynamically communicating tubes). In such cases, often particular hardware features are required to execute the method, for example a particular cache size or structure, particular shader units, a particular number of processor cores, or a particular kind of interconnect or communication means.

This situation often occurs in the field of computer arithmetic, where technical contributions are made at the very core of the computer (the arithmetic logic unit), but it may occur in other fields as well whenever special hardware is used to execute mathematical steps.

Second situation – adaptation of the method to the (known) computer

This is when the mathematical method is designed on the basis of technical considerations relating to the internal functioning of the computer, i.e. when the mathematical steps are specifically adapted to exploit the hardware on which they are implemented.

It is usually insufficient to define a generic technical implementation, and mere programming (of the generic computer) is not considered a technical task. However, if a mathematical method exploits particular hardware features of a (known) computer, it may contribute to a technical effect in terms of a technical implementation. For example, a mathematical method may be designed to utilize in a new and inventive manner the infrastructure available in a general-purpose computer (e.g., a CPU and a GPU, each with multiple cores, and each with independent memory, a network adapter, multiple levels of volatile and non-volatile memory/storage, …). If the thus-adapted method is

comprised as limiting features of the independent claims (G 1/9 §116), a contribution to a technical effect may be acknowledged.

It is insufficient for an algorithm to (merely) be more efficient than the prior art because any such benefit is inherent to the algorithm and is not a technical consideration underpinning its implementation.

A recurrent argument for protecting mathematical methods without a specific technical implementation and without an application to a technical field is that of improved computational efficiency (increased speed/less memory needed). Yet space/time complexity properties are inherent to any algorithm and cannot contribute to a technical character in a generic implementation.

However, if a technical contribution is present in one of the two dimensions, improved space/time complexity implies that the technical effect established by one of these dimensions is obtained with fewer resources and thus may contribute to technical character.

The idea that a generic implementation is not sufficient may be counter-intuitive, perhaps because any such implementation embodies the advantages of the algorithm in terms of space/time complexity, i.e. less processor time or less physical memory will be required in the real world to carry out the mathematical method.

These may appear to be very concrete technical advantages yet they may well only serve a non-technical purpose if no application is specified, meaning that there is no technical problem solved other than implementing the algorithm – which is straightforward under the assumption of a "generic implementation". As it is always possible to devise a method which solves the problem more slowly, the mere existence of a prior art method solving the same non-technical problem more slowly or inefficiently cannot bestow technical character on an otherwise non-technical method (T 1370/11). In other words, a non-technical method (such as a computer-implemented business method) does not become technical just because it is executed more efficiently.

A useful test of whether the computational savings are algorithmic (and hence not on the technical side in the sense of the EPC) or due to a specific technical implementation is the following. If the same savings of computational resources would occur if the algorithm were executed purely mentally by a team of mathematicians, or alternatively by any other computer hardware, then it is merely mathematical and inherent to the algorithm. It does not constitute a technical effect due to a specific technical implementation, but a non-technical mathematical effect. On their other hand, if the effect does not occur if executed mentally or only occurs on specific hardware, a technical effect due the technical implementation of the mathematical method may be present.

Mixed cases

Of course, the two situations may occur in intermingled cases. For example, a mathematical algorithm may be specifically adapted for load distribution. However, this effect only credibly occurs when the algorithm is executed on particular distributed-memory hardware with a particular interconnect. In such a case, neither the adaption of the algorithm is known, nor is the executing hardware part of a generic general-purpose computer. Both adaptations (of hardware and algorithm) may interact to contribute to a technical effect achieved by a particular technical implementation.

As before, the effect must be credibly achieved in substantially all embodiments, which makes it necessary for both adaptations to appear as limiting features in the independent claim.

Note that without an explicit limitation an effect related to a technical implementation can usually not be considered to be **implied**. For example, an algorithm "for load distribution" which exhibits advantageous effects on the executing hardware only when executed on parallel hardware may just as well be suitable to be executed on a single-core or serial processor. If so, this is a relevant embodiment, unless explicitly excluded by limiting features. In such a case, the desired effect is not achieved, also not implicitly.

Legal references:
G-II, 3.3.2; G 1/19

4. AI: the second dimension of the second hurdle – specific technical implementation

As for mathematical methods in general, the issue for methods involving AI is also whether the AI and ML method (or method steps) contributes to the technical character of the invention.

A claim can be deemed to be directed to a specific technical implementation if the AI algorithm is specifically adapted for that implementation or if the AI design is motivated by technical considerations regarding the internal functioning of the computer.

Example

Deep neural network convolutions mapped to graphics processing units (GPU) or an adapted multiply-accumulate unit.

It is usually insufficient to define a generic technical implementation, and mere programming (of the generic computer) is not considered a technical task.

It is also insufficient for an algorithm to (merely) be more efficient than the prior art because any such benefit is inherent to the algorithm and is not a technical consideration underpinning its implementation.

Legal references:
G-II, 3.3.1

5. Simulation, modelling and design

Computer-implemented methods of simulating, designing or modelling are examined according to the same criteria as any other computer-implemented inventions (G-VII, 5.4; G 1/19). A few special cases may be considered.

The simulation itself typically concerns a mathematical method. This may contribute to a technical effect, for example, in one of the following manners (G-II, 3.3.2; G 1/19).
▪ By interacting with the executing computer system via a technical implementation,
▪ By interacting with physical reality at the onset or throughout its execution via technical input,
▪ By providing technical output having a specific technical application or being limited to an intended technical use.

This list is not exhaustive, and other technical effects may be acknowledged as technology progresses. The individual cases are discussed in more detail below.

Simulations, modeling or design: technical implementations

A technical contribution that may be made by a model, simulation or algorithm because of their adaptation to the internal functioning of the computer system or network on which they are implemented is assessed in the same manner as adaptations of any other mathematical method to specific technical implementations (G-II, 3.3).

Simulations: technical input

An interaction with physical reality is not established by merely reading in data that has been measured. Whether data has been generated by a measurement or by some other method is not a property of the data itself in subsequent data processing (T 0489/14 R.7.3, T1615/17 R.2.4).

For example, a method step of "obtaining the value of a measured physical quantity" does not necessarily constitute technical input even if the value has been measured in the past (but not in the scope of the claim). However, if the method comprises a limiting step of using physical sensors to measure the instantaneous value of said physical quantity at a particular location, this constitutes technical input, because a technical measurement is a technical effect in the sense of the EPC.

If it is established that a simulation comprises technical input, it is to be assessed whether the mathematical steps in substantially all embodiments contribute to the technical effect of performing a technical measurement. This may be the case, for example, if the mathematical method forms part of an indirect measurement method that calculates the physical state of an existing real object. Typical cases include inferring values of physical quantities about the measured entity which have not been measured or even cannot be directly measured.

If, on the other hand, the mathematical method computes as output non-technical and/or abstract data, it does not contribute to the technical character. Examples include determining the value of a financial product based on measurements of the weather (T 1798/13), the determination of a usage fee for a forklift based on sensor measurements of its use (T 0199/16), and the automation of a business scheme based on measured information about airport closure (T 0288/19). The mathematical method of determining non-technical quantities solves a partial problem different from the technical partial problem solved by the technical measurement step, and the two sets of features are merely juxtaposed.

Simulations: technical output or intended technical use

The output of computer-implemented simulation methods usually consists of numerical data. Calculated numerical data reflecting the physical state or behaviour of a system or process existing only as a model in a computer usually cannot contribute to the technical character of the invention, even if it reflects the behaviour of the real system or process adequately. It is also not decisive whether the simulated system or process is technical or whether the simulation reflects technical principles underlying the simulated system.

A technical effect may, however, be achieved if the output is further used in an interaction with physical reality, for example by controlling a technical system. Calculated numerical data may have a "potential technical effect", which is the technical effect that will necessarily be produced when the

data is used according to an intended technical use. For this, the data must be **specifically adapted** for this intended use, and not have relevant non-technical uses (G-II, 3.3.2; G 1/19).

For example, a computer-implemented method may output control instructions for adjusting the temperature of a furnace in a machine-readable format suitable to automatically control the furnace. Even if the actual controlling is not claimed its potential technical effect is implied. On the other hand, if a computer-implemented method merely displays a predicted temperature curve of a model of a furnace, this information may be used for non-technical purposes, such as for gaining information about the furnace or for facilitating making cognitive decision-making, such as whether a prototype of the furnace should be built for economical reasons. In the latter case the technical effect is not achieved over substantially the whole scope of the claim (due to encompassing relevant non-technocal purposes) and therefore cannot be relied on in the assessment of inventive step

As elaborated above with respect to technical input, if it is established that there is technical output, it is to be assessed whether the mathematical steps in substantially all embodiments contribute to the technical effect achieved by this technical output. If they do not, the assessment of inventive step may reveal that two independent, juxtaposed partial problems are solved, one relating to the automation of mathematical steps, and the other to controlling a technical system or process.

Modelling and Design methods: technical output or intended technical use

The aforementioned principles apply equally if a computer-implemented simulation is claimed as part of a design process. The design process is normally a cognitive exercise, although it may be supported by mathematical steps and executed on a computer.

If a computer-implemented method results merely in an abstract model of a product, system or process, e.g. a set of equations, a geometric or visual description or another representation, this *per se* is not considered to be a technical effect, even if the modelled product, system or process is technical (G-II, 3.3.2; G 1/19).

The abstract representation of a process or product (model, design, etc) are mathematical in nature. It depends on the further use of the model, which must be specified at least implicitly in the claim, whether a technical effect can be considered in the assessment of inventive step.

From the mere generation of a design, usually no implicit technical use can be inferred, since designs often also have relevant non-technical uses, depending on the pertinent field of technology. Such uses may include visualisation, simulation for gaining scientific knowledge, tendering, costing, or the validation of administrative requirements. Models are also used for providing cognitive information to assist a user in mental processes, for example making the business decision whether to build a prototype (G 1/19). Most designs are never realised and so generating a design does not in itself imply its fabrication.

In such cases, in order to establish technical character, it may be necessary to limit the claim to an actual manufacturing step, or to the output of machine-readable control instructions providing a corresponding potential technical effect (i.e., a technical effect that **necessarily** occurs when the data is put to its intended use).

Mathematical steps relating to a design may, for example, contribute to the technical effect of manufacturing an item if the design steps purposively and deterministically bestow particular

technical properties on the item once manufactured, or if the design process modifies the manufacturing process itself (e.g., design for manufacturability).

Other contributions are conceivable, but they must occur in substantially all embodiments. They must also be deterministic, i.e., not subject to creative human decisions, lest the causal chain of technical character is broken.

The mere lowering of cognitive burden on the user, for example by visualizing a design or indicating locations requiring re-design, does not constitute a technical effect.

Modelling and Simulation methods: accuracy

Whether a simulation contributes to the technical character of the claimed subject-matter does not depend on the quality of the underlying model or the degree to which the simulation represents "reality" (G 1/19 §111).

Thus, generating a model or running a simulation that is more accurate that a prior art model or simulation does not bestow technical character on an otherwise non-technical method. The same argument holds as for computational efficiency: it is always possible to devise a less accurate method, so technical character cannot hinge *solely* on the accuracy of the mathematical method.

However, conversely, if a model or simulation is **not accurate enough** to credibly achieve a further technical effect (e.g., in controlling a technical system), the claimed modelling or simulation process may *in principle* contribute to technical character, but still may be considered non-inventive as the alleged improvement would not be achieved if the simulation is not accurate enough for its intended purpose.

That said, the mere fact that a model or simulation does not **perfectly** reflect reality (no model does) does not *per se* exclude a contribution to technical character. All that is required is that a technical effect is achieved in substantially all embodiments to which the mathematical method contributes, despite being inaccurate.

Legal references:
G-II, 3.3.2, G 1/19

6.   Examples for Simulation, modelling and design

Example 1 — first dimension of second hurdle – simulation method

The invention concerns a computer-implemented method for the virtual testing of a virtual weldment (T 2594/17).

Claim 1 comprises the step of "an analysis engine configured to perform **simulated testing of a 3D virtual weldment** (2200), and further configured to perform inspection of at least one of a 3D virtual weldment (2200) before simulated testing, a 3D animation of a virtual weldment (2200) under simulated testing, and a **3D virtual weldment** (2200) after simulated testing for at least one of pass/fail conditions and defect/ discontinuity characteristics; … wherein said **simulated testing** includes at least one of simulated destructive testing and simulated nondestructive testing of the **virtual weldment**."

FIG. 21
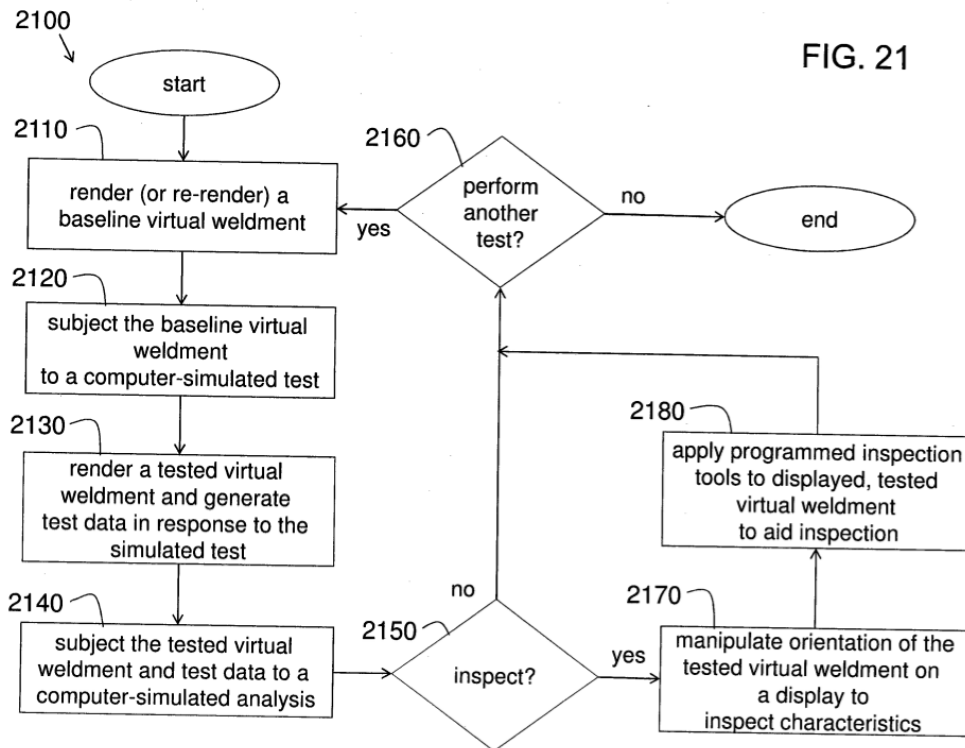
The mathematical method operates merely on a "virtual weldment" which need not exist in physical reality. The claim merely generates data which is not functionally limited to an inherently technical use but may be used for a variety of non-technical purposes, for example for training purposes. The board identified the computer implementation as the only technical feature and took the closest prior art to be a known general-purpose computer (R.3.1.3). The board held that the mathematical steps do not interact with the technical features of the claim (R.3.2.9) and thus do not contribute to the technical character of the invention; they merely perform image processing in order to generate cognitive data (R.3.2.5 and 3.2.11). The mathematical method is hence given to the skilled person as part of the problem to be solved (automating the mathematical method). The solution to this problem is straightforward, and merely requires routine programming skills (G-VII, 5.4).

This example shows that a known general-purpose computer can be used as closest prior art, as foreseen in G 1/19 §79.

Example 2 — first dimension of second hurdle – design method

The invention concerns "A computer-implemented method of developing a rod pattern design for a nuclear reactor" (T 2660/18). The method comprises a step of providing "data indicative of limits that were violated by the proposed test rod pattern design during the simulation".

The board held that, although the method yields a rod pattern design and provides limits of core performance values for a reactor plant having this design, this rod pattern design and the limits cannot be used directly in a nuclear reactor system. **The rod pattern would first need to be manufactured** (R.19).

Moreover, a rod pattern design has non-technical uses such as for study purposes. These are "relevant uses other than the use with a technical device" in the sense of G 1/19 (§94-95), and

therefore a technical effect is not achieved over substantially the whole scope of the claimed invention (R.19).

Example 3 — first dimension of second hurdle – design method

The invention concerns "wire harness design aiding method" (T 1371/16). The method comprises a step of "storing (S1) body data on an object to which a wire harness is installed, three-dimensional data on an auxiliary device installed on a vehicle body, and data on a minimum bending radius of the wire harness" and a step of "outputting (S9), if the wiring path does not satisfy the minimum bending radius, data on corrected wiring path data designed in light of the data on the minimum bending radius". The invention also comprised a corresponding apparatus.

FIG. 1
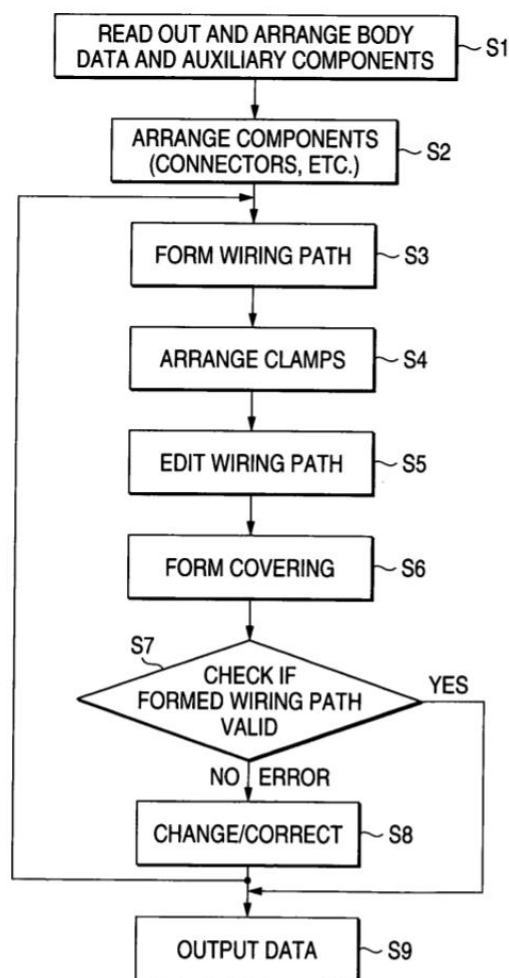
The board held that the purpose of the method is to output numerical data about the wiring path design but does not specify any further use of the output wiring path data, further properties or specific data format that could limit the possible uses of the data. In view of that, other relevant uses of the output data for non-technical purposes, for example informational, study or training purposes, are within the scope of the claim. Since the data can be output in any form or format, it cannot be considered specifically adapted for the purposes of an intended technical use (R.6.1).

This example shows that design data does not in itself imply a further use. Furthermore, even though the claim comprises "body data on an object to which a wire harness is installed", there is no limiting method step involving actually installing the wire harness, nor an implied use for this purpose.

Example 4 — indirect measurement

The invention concerns a "method of detecting human body temperature by scanning a temperature detector across a region of a forehead to measure a peak temperature reading of skin over a temporal artery and computing a body temperature as a function of the peak temperature based on a model of heat balance" (T 1985/16).
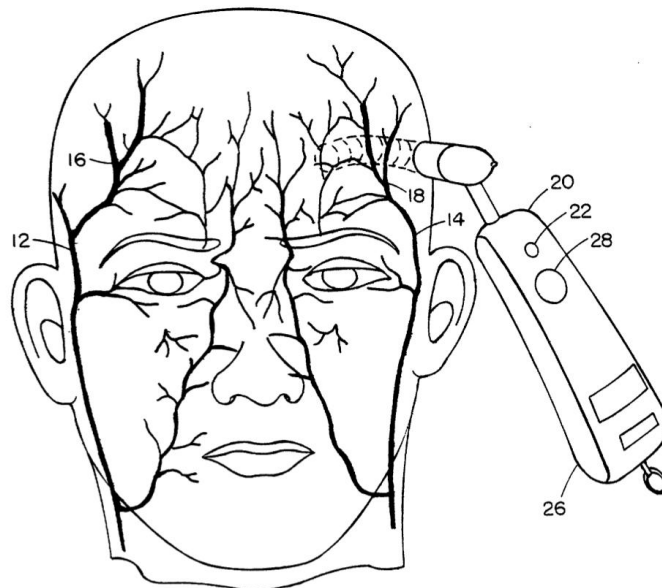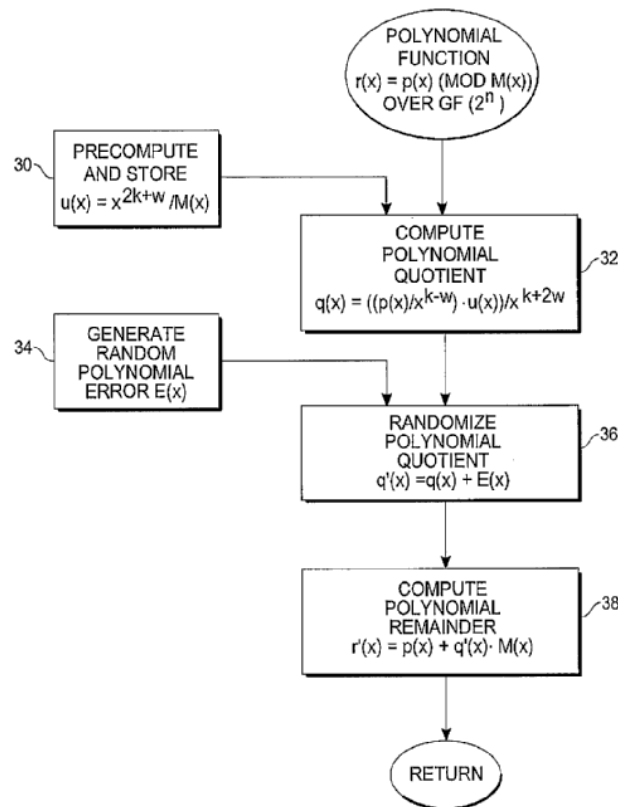

FIG. I

Even though this decision predates G 1/19, it is an example for the mathematical method ("a model of heat balance") contributing to an indirect measurement in the sense of G 1/19. Technical measurements are carried out on an existing physical object (the patient's forehead), and the mathematical method contributes to the technical character of the claimed method in that it results in the body temperature being provided (R.5.2.1), which is the physical state of a physical object (the same as measured).

Example 5 — second dimension of second hurdle

The invention concerns a "polynomial reduction operation" (T 1925/11). For a modulus of high degree (multi-word), the operation can be performed with word shifts rather than bit shifts. To this end, the formulae used are reformulated in terms of the "word size w", more precisely in terms of divisions by $x^{**}((2k+w))$ and $x^{**}((k-w))$.

POLYNOMIAL FUNCTION $r(x) = p(x)$ (MOD $M(x)$) OVER GF (2^n)

30 — PRECOMPUTE AND STORE $u(x) = x^{2k+w}/M(x)$

32 — COMPUTE POLYNOMIAL QUOTIENT $q(x) = ((p(x)/x^{k-w}) \cdot u(x))/x^{k+2w}$

34 — GENERATE RANDOM POLYNOMIAL ERROR $E(x)$

36 — RANDOMIZE POLYNOMIAL QUOTIENT $q'(x) = q(x) + E(x)$

38 — COMPUTE POLYNOMIAL REMAINDER $r'(x) = p(x) + q'(x) \cdot M(x)$

RETURN

Without going into the complex mathematical details of this example, the key message here is that the mathematical operations performed are specifically adapted to the underlying architecture that offers word shift operations.

From point 8 of the Reasons in the aforementioned case, it is clear that the board considers that the implementation of the algorithm in terms of word shifts (of the underlying hardware) contributes to inventive step, and thus implicitly to technical character. It is important to note that the claim is **implicitly limited** to using specific hardware capable of word shifts.

This is said to "simplify handling of the polynomial quantities on computational hardware".

Example 6 — second dimension of second hurdle – AI

In this patent (EP 1569128 B1), a machine learning engine is cleverly implemented on a specific hardware architecture comprising a CPU and GPU. Advantage is taken of the fast data processing characteristics of the GPU with its parallel data architecture and programmability.
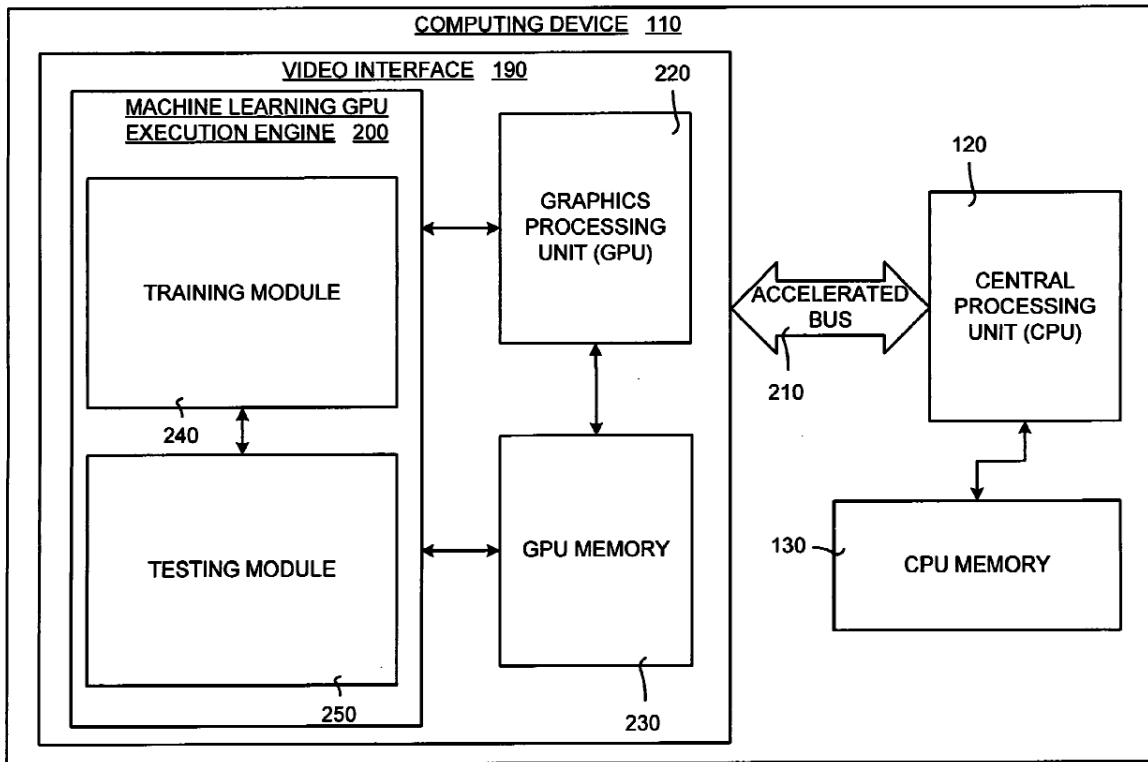
Example 7

Tasks are split such that preparatory steps are performed on the CPU and the more data-intensive training steps are performed on the GPU.
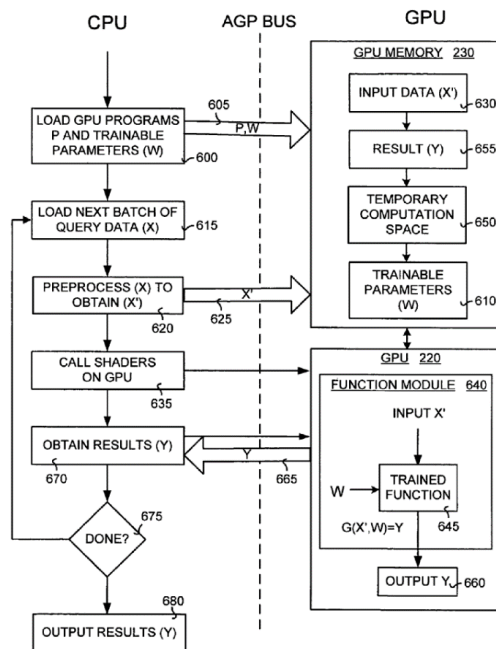
In this case, the implementation of machine learning takes into account the internal functioning of the computer in this case.